質問に対する回答



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\nabla \frac{1}{r} = \left( i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right)\frac{1}{r}$$

$$= i\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + j\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + k\frac{\partial}{\partial z}\left(\frac{1}{r}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = \frac{\partial}{\partial x}(r^{-1}) = -1\cdot r^{-2}\cdot\frac{\partial r}{\partial x} = -\frac{1}{r^2}\frac{\partial r}{\partial x}.$$

$$\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2} = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{\frac{1}{2}}$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}\cdot 2x$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$$\therefore \frac{\partial}{\partial x}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\cdot\frac{x}{r} = -\frac{x}{r^3}$$

以下同様に. $\frac{\partial}{\partial y}\left(\frac{1}{r}\right) = -\frac{y}{r^3}$, $\frac{\partial}{\partial z}\left(\frac{1}{r}\right) = -\frac{z}{r^3}$

以上まとめると

$$\nabla \frac{1}{r} = -\frac{x}{r^3}\mathbf{i} - \frac{y}{r^3}\mathbf{j} - \frac{z}{r^3}\mathbf{k}$$

$$= -\frac{1}{r^3}\underbrace{(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})}_{\text{位置ベクトル：}\mathbf{r}}$$

$$= -\frac{1}{r^3}\mathbf{r}$$

質問の内容

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{\partial r}{\partial x}$$ ← 何故これがつくのか？

$r(t)$ … 1変数~~関~~関数

$$\frac{d}{dt}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{dt}$$ と一緒.

第一章 合成関数の微分 …… $f(g(x))$ を $x$ で微分

$$\frac{df}{dx} = \frac{dg}{dx}\frac{df}{dg}$$

$$\frac{d}{dt}\left(\frac{1}{r_{(t)}}\right) = \frac{dr}{dt}\,\frac{d}{dr}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{dt}$$ … 合成関数の微分.

微分の定義に戻って考えると.

$$\frac{dr}{dt} \equiv \lim_{\Delta t \to 0} \frac{r(t+\Delta t) - r(t)}{\Delta t}$$

$$\frac{d}{dt}\left(\frac{1}{r}\right) = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left\{\frac{1}{r(t+\Delta t)} - \frac{1}{r(t)}\right\}$$

微分される関数が $r$ のを変数とする関数で, $r$ はさらに $t$ を変数とする関数

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t}\,\frac{r(t) - r(t+\Delta t)}{r(t+\Delta t)\,r(t)}$$

$$= \lim_{\Delta t \to 0} -\frac{1}{r(t)\cdot r(t+\Delta t)} \cdot \frac{r(t+\Delta t) - r(t)}{\Delta t}$$

$$= -\frac{1}{r^2_{(t)}}\frac{dr}{dt}$$

$$= -\frac{1}{r^2}\frac{dr}{dt}$$